



Stage long de recherche

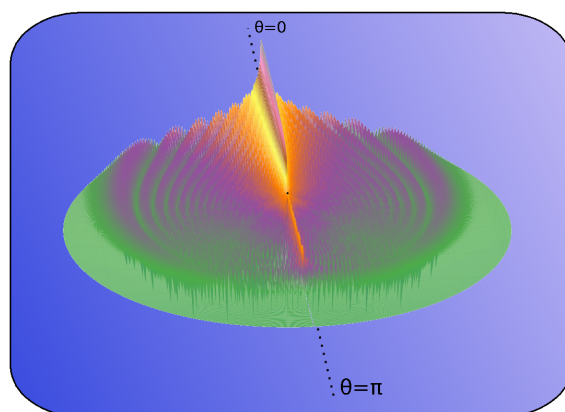
FIP M1

Étude des collisions entre atomes froids de rubidium et un gaz chaud

Study of collisions between cold rubidium atoms and a hot background gas

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Abstract

Le système étudié consiste en un ensemble d'atomes de rubidium refroidis dans un piège magnéto-optique puis transférés dans un piège magnétique quadrupolaire. Nous avons cherché à caractériser l'effet d'un gaz résiduel ($10^{-8} - 10^{-9}$ Torr) à température ambiante sur ce système. Une collision avec un atome chaud peut causer la perte d'un atome piégé, mais pour les grands paramètres d'impact le transfert d'énergie est insuffisant pour que l'atome sorte du piège. Ainsi, on observe à la fois une perte d'atomes du piège et l'augmentation de l'énergie moyenne des atomes restants. Nous avons mesuré ce réchauffement expérimentalement en sondant la distribution d'énergie des atomes à l'aide d'ondes radio. De plus nous avons mis au point une simulation directe du piège tenant compte du mouvement des atomes et des collisions avec le gaz chaud. Ces collisions sont régies par la section efficace différentielle calculée numériquement à partir de premiers principes. La comparaison entre expérience, simulation et des modèles simples doit confirmer que les principaux mécanismes à l'œuvre sont bien compris et qu'ils peuvent être mesurés et prédits.

We have studied an ensemble of rubidium atoms cooled in a magneto-optical trap and transferred into a magnetic quadrupole trap. The goal was to characterize the effects of a room-temperature residual background gas ($10^{-8} - 10^{-9}$ Torr) on this system. A collision between a trapped atom and a background particle can cause the former to be lost from the trap, but for glancing collisions this is not always the case: the energy transferred to the trapped atom might not be sufficient for it to leave the trap. Therefore, as atoms are lost from the trap, we also expect to see the average energy of the remaining atoms increase. This heating was experimentally measured using a radio-frequency field to probe the energy distribution in the trap. Moreover, a direct numerical simulation of the trap was designed. It takes into account the classical motion of atoms inside the trap and collisions with background atoms. These collisions are governed by the differential cross-section, which was calculated numerically from first principles. Comparing experiment, simulation and simple theoretical models should confirm that the mechanisms at work are well understood and that they can be measured and predicted.

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Introduction

This report sums up the work I have been involved in during a 5 month stay in the Quantum Degenerate Gas laboratory (QDG) at the University of British Columbia (UBC) in Vancouver. This internship was supervised by Dr Kirk Madison from UBC and Dr James Booth from the British Columbia Institute of Technology (BCIT).

The main subjects of interest in the QDG lab are

- the creation of cold polar molecules in an optical lattice,
- the characterization of loss and heating in pure-magnetic and magneto-optical traps.

I was involved in the latter project.

Interest in loss and heating processes in atom traps arose when contemplating the feasibility of a miniature atom trap. The lifetime of atoms in a trap is limited by collisions with hot atoms from the residual background vapour. Such collisions often impart enough kinetic energy to the trapped atoms for them to escape the trapping potential. Therefore a good vacuum (on the order of 10^{-8} Torr) is indispensable to trap atoms for more than a few seconds. However pumping systems

take up a lot of space, so a miniature trap might have to do without one. The plan was to trap atoms in a sealed cell under high vacuum. But it appeared that gases released from the cell walls prevented the formation of long-lived traps.

Nevertheless, this attempt drew attention to the fact that some background gases affect atom traps more than others. Indeed, the loss rate from the trap depends on the collisional cross-section between trapped atoms and background atoms. Therefore trap loss is more than just a hindrance: it provides a tool to investigate cross-sections. Collisions range from glancing collisions with a large impact parameter to very energetic head-on collisions. Given a trap depth, only some of these collisions will contribute to loss. Measuring loss rates for various trap depths is therefore a way of characterizing this distribution of collisions. This was done prior to my arrival by studying the evolution of trapped 87 rubidium in the presence of a background vapour of 40 argon [8].

After having characterized losses from a magnetic trap, the team's focus shifted to heating. The glancing collisions mentioned above do not involve a large enough energy transfer to cause the loss of the trapped particle, but they do affect that particle's energy. The overall effect of such collisions is an increase of the average energy per particle in the trap¹.

Before we start, let me sum up the main goals that were set during his internship:

- experimentally observe and quantify heating in a magnetic trap,
- predict and observe the dependence of heating with respects to trap depth, background pressure and the temperature of the trapped atoms
- and create an accurate direct numerical simulation of the system, compare its results with the experiment and use it to make predictions.

Although I have participated in the pursuit of all of these goals, the main focus of my work was the last objective: simulating the system to get a better understanding of it.

1 Theoretical background

The object of this section is to provide a quick overview of the concepts I will be dealing with further on in this report. It is by no means exhaustive, please look-up the references for more details.

¹I might refer to this average energy per particle as a temperature in this report ($E = k_B T$, with k_B the Boltzmann constant). This should be seen as a convenient change of units rather than as a statement on the thermodynamics of the system. Indeed, our system is way out of equilibrium and the strict definition of temperature does not apply.

1.1 Trapping neutral atoms

First of all, I will describe the principles behind the trapping of atoms using light and/or magnetic fields.

1.1.1 Magnetic trap

In an atom, the nuclear and electronic spins ($\mathbf{I}$ and $\mathbf{S}$) and the electronic orbital angular momentum ($\mathbf{L}$) all contribute to an atomic magnetic moment $\boldsymbol{\mu}$. Its amplitude is described by the quantum number F . In a magnetic field, the projection of this moment along the field takes quantized values $g_F \mu_B m_F$, where g_F is the *Landé g-factor*², μ_B is the Bohr magneton and $m_F = -F, -F + 1, \dots, F$. The energy of an atom in a magnetic field $\mathbf{B}$ is shifted by $\Delta E = -\boldsymbol{\mu} \cdot \mathbf{B}$. Therefore atoms with the same F but different m_F numbers will be degenerate in energy if there is no magnetic field and that degeneracy will be lifted in the presence of a field. This is referred to as Zeeman splitting.

The states for which $\boldsymbol{\mu}$ and $\mathbf{B}$ are aligned will have a lower energy in higher fields (high-field-seeking states) whereas those for which $\boldsymbol{\mu}$ and $\mathbf{B}$ are anti-aligned will have a lower energy in smaller fields (low-field-seeking states). In free-space a magnetic field maximum cannot be achieved, but a local minimum can, and this minimum will be seen as a potential well by low-field-seeking atoms.

It should be noted here that the direction of the field is not necessarily uniform. When an atom moves in an inhomogeneous field, its magnetic moment will follow the local magnetic field so long as the variations of the field are not too fast. In that case the motion is said to be *adiabatic*. However if the magnetic field changes quickly, the atom cannot follow and a trapped, anti-aligned state might suddenly find itself aligned and be lost from the trap (figure 1).

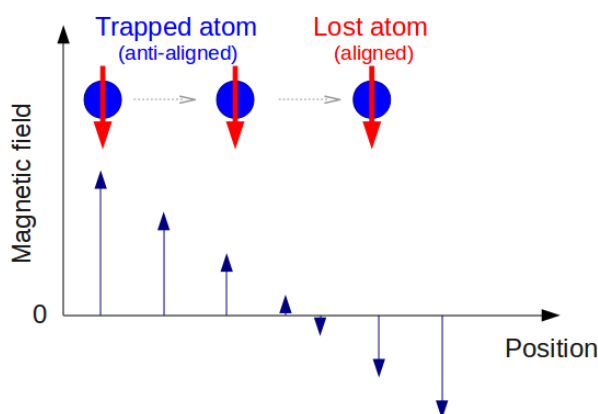


Figure 1: A case of Majorana spin-flip: the atom crosses the magnetic field zero and goes from being anti-aligned (trapped) to aligned (lost).

²The Landé g-factor is a dimensionless proportionality constant that depends on I , L and S .

This type of *adiabatic* process is called a *Majorana spin-flip*, it is liable to happen if the rate of change of the magnetic field (due to the atom's motion) becomes of the same order as the frequency associated with the transition [11]:

$$\mathbf{v} \cdot \frac{\nabla B}{B} \sim \frac{\boldsymbol{\mu} \cdot \mathbf{B}}{\hbar}, \quad (1.1)$$

The simplest setup to create a magnetic field minimum consists of two coils in anti-helmholtz configuration. That is two coils aligned along the z-axis, separated by a distance D and carrying a current I running in opposite directions. These two magnetic dipoles with opposite directions form a magnetic quadrupole, hence the name quadrupole trap.

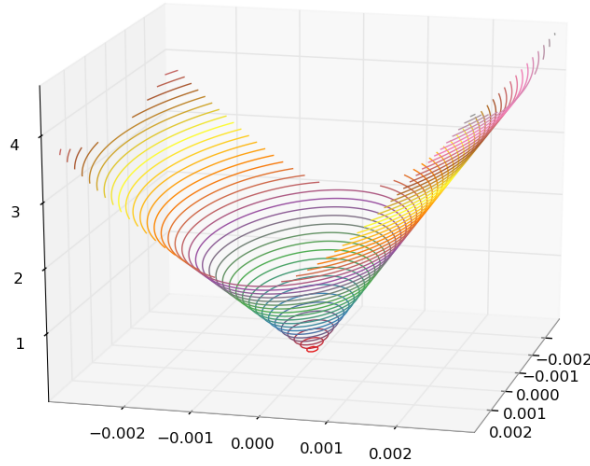


Figure 2: Shape of the potential seen by an atom in the xz-plane of a magnetic quadrupole trap. The potential increases linearly with the coordinate $c = \sqrt{x^2 + y^2 + (2z)^2}$.

Near the centre of the trap, the magnetic field amplitude varies linearly with distance from the minimum. The radial field gradient is half the axial gradient. The magnetic field goes to zero and changes sign in the centre (figure 2), which raises the question of Majorana losses. If $B = b'x$, with x a position coordinate, the condition given in equation (1.1) becomes:

$$x^2 \sim \frac{\hbar}{\mu b'} \quad (1.2)$$

This defines the surface through which an atom has to pass to undergo a spin-flip. The associated loss rate is:

$$\Gamma_{Majorana} \sim nx^2v \sim n \frac{\hbar v^2}{\mu b'} \sim n \frac{\hbar k_B T}{m \mu b'} \quad (1.3)$$

where n is the density, v is a typical speed and $k_B T$ is an energy scale for trapped atoms. In typical conditions for our experiment, this rate is on the order of 10^{-7} s^{-1} (versus 10^{-1} s^{-1} for collisional losses), so we will neglect Majorana losses.

To the linear potential felt by low-field-seeking atoms because of their interaction with the magnetic field, one must add the gravitational potential (though it is relatively small in most cases). The trap depth is generally limited by the glass walls of the vacuum cell: atoms that reach the room-temperature walls are instantly heated and lost from the trap.

1.1.2 The radio-frequency knife

The Zeeman splitting between different m_F states is in the radio-frequency (RF) range. Therefore, an RF field can cause magnetic dipole transitions between these states [5]. But we have seen in the last paragraph that only certain m_F states are trappable (*i.e.* are low-field-seeking), so the RF can be used to eject atoms from the trap. Since the magnetic field amplitude is position dependent, so is the energy splitting between different states. As a consequence the RF will only be resonant with a transition at a precise position, that is, at a given potential energy. The RF frequency ν defines a closed surface around the centre of the trap, and all particles that cross that shell are lost from the trap. Since only particles with a total energy, potential and kinetic, larger than $h\nu$ can reach that shell, and assuming that they do so in at most a few hundred microseconds, the RF can be used to empty out all atoms with energies $h\nu$ or larger (figure 3). Its ability to “cut” energy distributions has earned this technique the name of “RF knife”. Of course, the RF knife can also be used to set the trap depth.

1.1.3 Magneto-optical trap

The magnetic trap can only trap atoms that are already cold, therefore, it has to be loaded from another type of trap that actively captures and cools atoms: a magneto-optical trap (MOT).

When photons scatter off an atom, they transfer some momentum to that atom, *i.e.* the atom experiences a force (called radiation pressure). A magneto-optical trap ingeniously uses that force to capture and trap atoms.

The scattering of photons off atoms can be seen as a process of absorption and re-emission. When the photon is absorbed, it transfers its momentum $\hbar\mathbf{k}$ to the atom. When the atom spontaneously re-emits, it recoils by $-\hbar\mathbf{k}^*$, where $\mathbf{k}^*$ is the wave-vector of the emitted photon, which is equal in amplitude to $\mathbf{k}$ but is random in direction. If an atom scatters a great number of photons coming from one direction, the absorptions will all push the atom in the same direction, but the effect of the emissions will average out. Thus the atom experiences a net force in the direction of $\mathbf{k}$ (figure 4).

The rate at which a two-level atom scatters photons is given by the lorentzian [7, 14]:

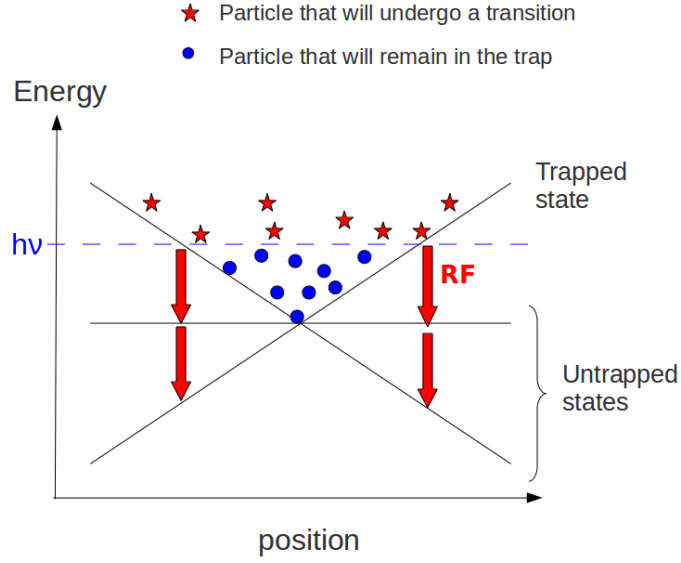


Figure 3: The RF knife transitions atoms of a given potential energy to an untrappable state, causing them to be lost from the trap. We assume that all atoms with a total energy above $h\nu$ have undergone a transition after a few hundred milliseconds. The energy levels shown here correspond to an atom in the $F = 1$ state ($m_F = -1, 0, +1$).

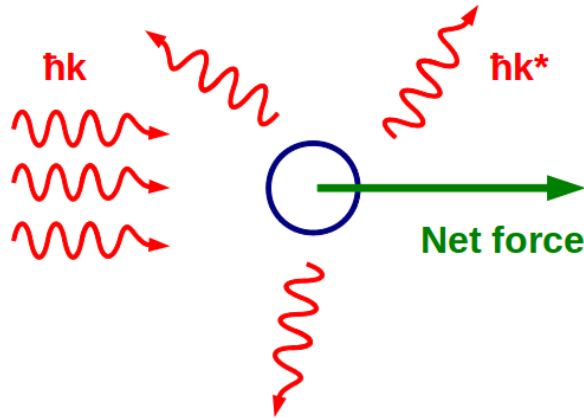


Figure 4: The net force on an atom scattering photons is in the direction of the incoming light. Indeed the momenta transferred to the atom add up whereas the recoils due to spontaneous emission cancel out.

$$r = \frac{\gamma}{2} \frac{s_0}{1 + s_0 + (2\delta/\gamma)^2} \quad (1.4)$$

where γ is the spontaneous emission rate from the upper state and $s_0 = I/I_{sat}$ is the saturation parameter, ratio of the light intensity I and the saturation intensity I_{sat} , which depends on the properties of the atom. The detuning $\delta = \omega - \omega_0$ is the difference between the light's angular frequency and the frequency associated with the energy gap in the two level atom.

Since each scattering event transfers a momentum $\hbar\mathbf{k}$ to the atom on average, the radiation pressure³ is given by:

$$\mathbf{F} = \frac{\gamma}{2} \frac{s_0}{1 + s_0 + (2\delta/\gamma)^2} \hbar\mathbf{k} \quad (1.5)$$

At a given intensity, this force decreases when the absolute value of the detuning increases.

A moving atom will see a Doppler-shifted light frequency: if the atom is travelling at v in the direction of the light's propagation,

$$\omega = \omega_l \left(1 - \frac{v}{c}\right) \quad (1.6)$$

Where ω_l is the light's frequency in the lab frame.

Therefore the detuning is a function of the atom's velocity: if $\delta_l = \omega_l - \omega_0$,

$$\delta = \omega_l \left(1 - \frac{v}{c}\right) - \omega_0 = \delta_l - kv \quad (1.7)$$

An atom lit by a red-detuned laser⁴ will scatter more photons if it is travelling towards the light source ($v < 0$) than if it is travelling away from it ($v > 0$).

Now if red-detuned light is shone on an atom from two opposite directions, the atom will experience two opposing forces, but the strongest of the two will always oppose his velocity. Indeed, the resultant force is:

$$F_{res} = \frac{\gamma}{2} \left(\frac{s_0}{1 + s_0 + (2(\delta_l - kv)/\gamma)^2} - \frac{s_0}{1 + s_0 + (2(\delta_l + kv)/\gamma)^2} \right) \hbar k \quad (1.8)$$

$$F_{res} \simeq \frac{8\hbar k^2 s_0 \delta_l / \gamma}{(1 + s_0 + (2\delta_l/\gamma)^2)^2} v \quad (1.9)$$

Since δ_l is negative, this is a friction force. This effect is called Doppler cooling. If six red-detuned laser beams are shone in the $\pm x, \pm y, \pm z$ directions, atoms will be slowed whatever the direction they are travelling in: we have a so called 3D *optical molasses*.

Atoms that enter the molasses experience a force which depends on their velocity but has no spatial dependence. When an atom's velocity nears zero, the friction

³It is really a force.

⁴A red-detuned laser has a frequency $\omega_l < \omega_0$ ($\delta_l < 0$), a blue detuned laser has a frequency $\omega_l > \omega_0$ ($\delta_l > 0$).

force also goes to zero and the random recoils due to spontaneous emission cause the atom to undergo a random walk in momentum space, eventually leading it out of the molasses. This is a typical example of Brownian motion. A consequence of this is the Doppler limit to cooling which is reached when the diffusion of the momentum prevents any further cooling⁵ [9, 14].

To introduce a spatial dependence to the radiation pressure, an inhomogeneous magnetic field is applied. As we have seen in section 1.1.1, an atom's energy levels are slightly shifted in a magnetic field due to the Zeeman effect, and that shift depends on the quantum number m_F and on the magnetic field amplitude. The energy difference between a $|F, m_F\rangle$ ground state and a $|F', m'_{F'}\rangle$ excited state is therefore a function of the local magnetic field, and so of the position. Actually, if the magnetic field is a quadrupolar field (as described in section 1.1.1), the magnetic field amplitude increases linearly with position⁶, and so does the Zeeman splitting.

To illustrate, let us consider the transition from a $F = 0$ ground state ($m_F = 0$) to a $F' = 1$ excited state ($m'_{F'} = -1, 0, +1$)⁷. Polarized light can be used to drive only certain transitions. Left-circularly-polarized light (denoted σ^-) only allows transitions with $m'_{F'} = m_F - 1$ whereas right-circularly-polarized (σ^+) light only allows $m'_{F'} = m_F + 1$ transitions. Suppose σ^- light is shone from the side where the Zeeman shift brings the $m'_{F'} = -1$ state closer to resonance (let this be the right-hand side, as in figure 5) and σ^+ is shone from the opposite direction (the left-hand side in figure 5). An atom on the right of the magnetic zero will more likely undergo the near-resonant transition to $m'_{F'} = -1$ and therefore absorb more σ^- photons and be pushed back to the centre of the trap. Conversely an atom in the left-hand side region will absorb more σ^+ photons that allow it to make the near-resonant transition to $m'_{F'} = +1$.

This clever combination of magnetic field and circularly-polarized light creates the missing position-dependent force (figure 6): now the MOT can capture atoms from a vapour, cool them and trap them.

1.2 Collisions with the background gas

1.2.1 Classical collision kinematics

Consider two colliding particles of masses m_1 and m_2 and of velocities $\mathbf{v}_1$ and $\mathbf{v}_2$. They interact through a short-range potential $V(\mathbf{r})$, where $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ is the vector that joins the two particles. The collision being a brief and violent event, we can suppose that these two particles are isolated (the effect of exterior forces is negligible in such a short time).

⁵Another source of heating is the irregularity with which photons are absorbed, which leads to “jiggling” of the atoms

⁶Near the centre of the trap.

⁷This scheme also works in more complex cases.

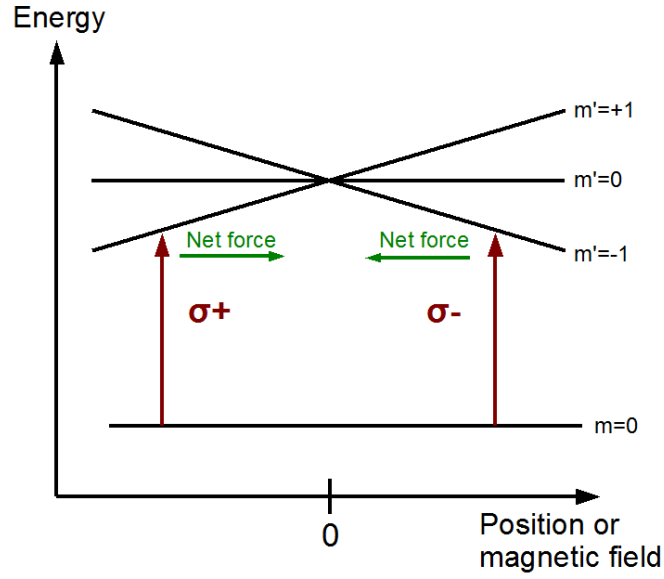


Figure 5: The non-uniform magnetic field creates a position-dependent trapping force by bringing atoms closer to resonance with the light that is pushing them towards the centre of the trap.

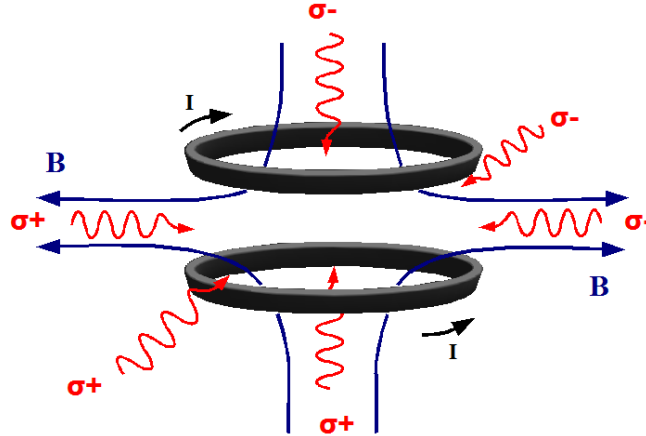


Figure 6: Diagram of a magneto-optical trap showing the six contra-propagating beams and their polarizations, as well as the coil currents and magnetic field lines.

To solve this two-body problem we introduce the usual quantities [3, 4]: the velocity of the centre of mass, the relative velocity and the reduced mass:

$$\mathbf{v}_{cm} = \frac{m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2}{m_1 + m_2} \quad (1.10)$$

$$\mathbf{v}_r = \mathbf{v}_1 - \mathbf{v}_2 \quad (1.11)$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \quad (1.12)$$

Conservation of momentum leads to:

$$\mathbf{v}_{cm}^i = \mathbf{v}_{cm}^f \quad (1.13)$$

Where the exponents i and f refer respectively to the initial and final states of the system.

The movement of the centre of mass being dealt with, the problem is reduced to the motion of a pseudo-particle of mass μ , position $\mathbf{r}$ and velocity $\mathbf{v}_r$ in the potential $V(\mathbf{r})$.

When the particles are far apart their interaction energy is equal to zero, all their energy is kinetic energy, so the conservation of energy between the initial and final states imposes:

$$|\mathbf{v}_r^i| = |\mathbf{v}_r^f| \quad (1.14)$$

If $\mathbf{v}_r^i$ is taken to be the z-axis of a spherical coordinate system, the direction of $\mathbf{v}_r^f$ is defined by an inclination angle θ and an azimuthal angle ϕ . Once the initial velocities and the angles θ and ϕ are known, the post-collision velocities can be calculated:

$$\mathbf{v}_1^f = \mathbf{v}_{cm} + \frac{\mu}{m_1} \mathbf{v}_r^f \quad (1.15)$$

$$\mathbf{v}_2^f = \mathbf{v}_{cm} - \frac{\mu}{m_2} \mathbf{v}_r^f \quad (1.16)$$

To calculate these collision angles the details of the interaction between the particles are needed, as well as the spatial configuration of the initial state. To eliminate that spatial dependence the notion of differential cross-section is introduced. Instead of considering the outcome of one pseudo-particle scattering off a potential, we are going to consider a uniform flux of such particles, and look at the distribution of angles of the outgoing particles.

For an incident flux $\mathcal{F}$, if dN is the number of particles that leave the region of interaction in the solid angle $d\Omega$ per unit time, the *differential cross-section* $\frac{d\sigma}{d\Omega}$ is defined by:

$$dN = \mathcal{F} \frac{d\sigma}{d\Omega} d\Omega \quad (1.17)$$

The total cross-section is defined as the integral of the differential cross-section over all solid angles:

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega \quad (1.18)$$

It has the units of an area: classically it can be seen as the effective surface the target particle presents to the incoming flux. Note that it generally is a function of the relative speed $\mathbf{v}_r$.

1.2.2 Calculation of the differential cross-section

The differential cross-section can be obtained by using a simple quantum-mechanical model.

As in the classical case, the quantum two-body problem can be reduced to solving for a pseudo-particle of mass μ scattering off a potential $V(\mathbf{r})$ [5, 4]. The Hamiltonian of the system is:

$$\hat{H} = \frac{\hat{\mathbf{p}}_r^2}{2\mu} + V(\hat{\mathbf{r}}) \quad (1.19)$$

The time independent solutions of Schrödinger's equation verify:

$$\left[-\frac{\hbar^2}{2\mu} \Delta + V(\mathbf{r}) \right] \psi(\mathbf{r}) = E\psi(\mathbf{r}) \quad (1.20)$$

Where E is the energy of the system.

Suppose that $V(r)$ tends to zero faster than $1/r$ as r goes to infinity, *i.e.* the interaction takes place in a limited volume (this is verified by a typical Lennard-Jones potential $V(r) = \frac{C_{12}}{r^{12}} - \frac{C_6}{r^6}$). In that case, the potential term can be neglected at large r . Therefore the total energy of the system, E , is the incoming particle's initial kinetic energy and the wave function at large r is a solution of Schrödinger's equation for a free particle.

Remember that our goal here is to obtain the probability distribution for the angles θ and ϕ given a uniform incident flux. This prompts us to write the wave function at large r as the sum of an incoming plane wave and a scattered spherical wave, which are both solutions of the free-particle equation (figure 7):

$$\psi(\mathbf{r}) \underset{r \rightarrow \infty}{\sim} N \left(e^{ikz} + f(k, \theta, \phi) \frac{e^{ikr}}{r} \right) \quad (1.21)$$

Where N is a normalization factor⁸ and $k = \frac{\sqrt{2\mu E}}{\hbar}$.

⁸We are leaving the problem of normalization aside, as the constant N will cancel out anyway.

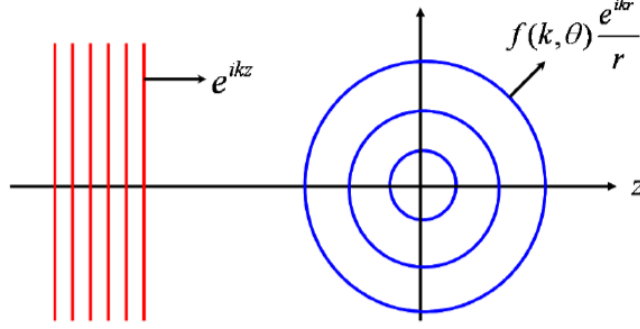


Figure 7: The wave function is supposed to tend asymptotically to the sum of two solutions of the free Schrödinger equation: an incoming plane wave and a scattered spherical wave.

The probability current density associated to that wave function sheds light on the reasons behind that choice. It is defined by [5, 4]

$$\mathbf{j}(\mathbf{r}) = \frac{\hbar}{2i\mu} (\psi(\mathbf{r})^* \nabla \psi(\mathbf{r}) - \psi(\mathbf{r}) \nabla \psi(\mathbf{r})^*) \quad (1.22)$$

The current density associated with the plane wave is:

$$\mathbf{j}_{in}(\mathbf{r}) = |N|^2 \frac{\hbar k}{\mu} = |N|^2 v_r \quad (1.23)$$

It has the form of a uniform flux of particles with speed v_r .

The current associated with the spherical wave is:

$$\mathbf{j}_{out}(\mathbf{r}) = |N|^2 \frac{\hbar k}{\mu} \frac{|f(k, \theta, \phi)|^2}{r^2} \quad (1.24)$$

The current through an element of surface $r^2 d\Omega$ is $|N|^2 \frac{\hbar k}{\mu} |f(k, \theta, \phi)|^2 d\Omega$. Thus the fraction of incident flux which is deflected into the solid angle $d\Omega$ is $|f(k, \theta, \phi)|^2 d\Omega$.

By definition of the differential cross-section,

$$\frac{d\sigma}{d\Omega} = |f(k, \theta, \phi)|^2 \quad (1.25)$$

At that point, let us make another hypothesis: $V(\mathbf{r}) = V(r)$ is a central potential. Again, the Lennard-Jones potential satisfies that condition. Then the problem has cylindrical symmetry around the z -axis (as defined in figure 7). Therefore, the wave function $\psi(\mathbf{r})$ is independent of the azimuthal angle ϕ and it can be expanded in a Legendre series:

$$\psi(r, \theta) = \sum_{l=0}^{+\infty} R_l(r, \theta) P_l(\cos \theta) \quad (1.26)$$

where P_l is the l^{th} Legendre polynomial. This is known as the *partial wave expansion* [5, 4]. The l^{th} partial wave has an angular momentum l . To satisfy the boundary condition given in equation (1.21), it can be shown that R_l must tend asymptotically to:

$$R_l(r, \theta) \underset{r \rightarrow \infty}{\sim} \frac{1}{kr} (2l+1) i^l e^{i\delta_l} \sin(kr - l\frac{\pi}{2} + \delta_l) \quad (1.27)$$

Where δ_l is a real constant called the phase shift of the l^{th} partial wave. The scattering amplitude $f(k, \theta)$ can then be written:

$$f(k, \theta) = \frac{1}{k} \sum_{l=0}^{+\infty} (2l+1) i^l \sin \delta_l P_l(\cos \theta) \quad (1.28)$$

We can then go about solving Schrödinger's equation to find the phase shifts δ_l . How this is done numerically has been detailed in [7] using a technique described in [10].

1.3 Losses and heating

1.3.1 Trap loss

Consider the collisions of a trapped atom of initial velocity $\mathbf{v}_t$ with background atoms of initial velocity $\mathbf{v}_b$ ⁹. The trapped atom sees a flux of such background particles equal to:

$$\mathcal{F}(\mathbf{v}_b) = n_b d(\mathbf{v}_b) (\mathbf{v}_b - \mathbf{v}_t) \quad (1.29)$$

where n_b is the background gas density (supposed to be constant in the cell) and $d(\mathbf{v}_b)$ is the probability distribution function for the background particle's velocity, *i.e.* the Maxwell-Boltzmann distribution at $T = 300$ K.

The fraction of that flux that will end up making an angle between θ and $\theta + d\theta$ with the direction of the flux is:

$$\int_0^{2\pi} \left(\frac{d\sigma}{d\Omega} \sin \theta d\theta \right) d\phi = 2\pi \left| f \left(k = \frac{\mu v_r}{\hbar}, \theta \right) \right|^2 \sin \theta d\theta \quad (1.30)$$

Only some of these collision angles correspond to a final velocity for the trapped particle that will make it leave the trap.

To simplify, let us suppose that the trapped particle is initially static, *i.e.* $\mathbf{v}_t^i = \mathbf{0}$. In that case $\mathbf{v}_r^i = \mathbf{v}_b^i$, $\mathbf{v}_{cm} = \frac{m_b}{m_b + m_t} \mathbf{v}_b^i$ and equation (1.16) becomes

$$\mathbf{v}_t^f = \frac{m_b}{m_b + m_t} (\mathbf{v}_r^i - \mathbf{v}_r^f)$$

⁹I am keeping the same notations as in the previous section, except the index 1 is replaced by b , for background particle, and the index 2 is replaced by t , for trapped particle. I am also dropping the exponent i , all velocities referred to being initial velocities, unless otherwise specified.

So the energy transferred by a collision of angle θ to the trapped atom is:

$$\Delta E = \frac{\mu^2}{m_t} v_b^{i2} (1 - \cos \theta) \quad (1.31)$$

If U is the trap depth, a collision will expel the atom out of the trap if [1, 2, 8]:

$$\Delta E > U \quad (1.32)$$

$$\text{or } \theta > \theta_{min}, \quad (1.33)$$

$$\text{where } \theta_{min} = \arccos \left(1 - \frac{U m_t}{\mu^2 v_b^{i2}} \right) \quad (1.34)$$

If we define the partial loss cross-section:

$$\sigma_U(v_b) = 2\pi \int_{\theta_{min}}^{\pi} \left| f \left(\theta, k_r = \frac{\mu v_b}{\hbar} \right) \right|^2 \sin \theta d\theta, \quad (1.35)$$

the rate at which the trapped atom is being driven out of the trap by particles of velocity $\mathbf{v}_b$ is [8]:

$$\mathcal{F}(\mathbf{v}_b) \sigma_U(v_b) = n_b d(\mathbf{v}_b) v_b \sigma_U(v_b) \quad (1.36)$$

To obtain the total loss rate for a trapped atom, the previous expression must be integrated over the background velocities:

$$\Gamma_U = n_b \iiint d(\mathbf{v}_b) v_b \sigma_U(v_b) d\mathbf{v}_b = n_b \langle \sigma_U v_b \rangle \quad (1.37)$$

For simplicity, we will only write

$$\Gamma = n_b \langle \sigma v \rangle \quad (1.38)$$

when this is not ambiguous. $\langle \sigma v \rangle$ is known as the loss *rate coefficient*.

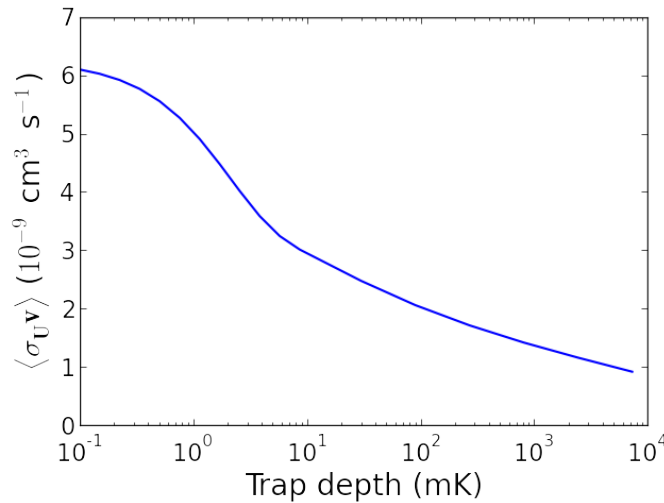


Figure 8: Theoretical loss rate coefficient $\langle \sigma_U v \rangle$ versus trap depth U for the collision of 300 K rubidium atoms with static rubidium atoms.

1.3.2 Trap population

If several background gases are present, the loss rate is the sum of their contributions.

$$\Gamma = n_{b1}\langle\sigma v\rangle_1 + \dots + n_{bi}\langle\sigma v\rangle_i \quad (1.39)$$

In the magnetic trap nothing compensates for this loss and the number of atom exponentially decays:

$$N_{MT}(t) = N_{MT}(0)e^{-\Gamma t} \quad (1.40)$$

In the MOT, however, the atoms are actively being captured at a rate R . Moreover the presence of excited state atoms with large cross-sections leads to *light-assisted* collisions between trapped atoms [16]. The losses associated to these collisions are characterized by the constant β and are proportional to the overlap of the cloud with itself. The equation for the trap population is therefore:

$$\frac{dN_{MOT}}{dt} = R - \Gamma N_{MOT} - \beta \int n^2 dV \quad (1.41)$$

Where n is the density in the trap. In first approximation we can neglect light-assisted collisions and the equation can be solved (for an initial population of zero):

$$N_{MOT}(t) = \frac{R}{\Gamma} (1 - e^{-\Gamma t}) \quad (1.42)$$

1.3.3 Heating

As we have seen earlier, not all collisions lead to loss. The total collision rate is also the loss rate at zero trap depth (in which case any collision expels the atom from the trap), so per unit time, each trapped atom undergoes $n_b\langle\sigma_0 v_b\rangle - n_b\langle\sigma_U v_b\rangle$ collisions which do not lead to loss. These atoms are promoted to higher energy orbits and therefore the average energy of atoms in the trap increases. This is what we will refer to as heating.

The heating rate for static atoms in a trap of depth U is given by [1, 2]:

$$Q_U = n_b\langle v_b 2\pi \int_0^{\theta_{min}} \Delta E(\theta) \left| f\left(\theta, k_r = \frac{\mu v_r}{\hbar}\right) \right|^2 \sin\theta d\theta \rangle \quad (1.43)$$

Where $\langle.. \rangle$ still refers to averaging over the Maxwell-Boltzmann distribution and $\Delta E(\theta)$ is the energy transfer for a collision of angle θ (note that it is also a function of v_r , see equation (1.31)).

At this point the approximation of static trapped atoms breaks down: even if they start at a very low temperature, atoms eventually acquire an energy which is no longer negligible compared to the trap depth. The way they are lost from the trap or heated further can no longer be described accurately by our model.

One purpose of the N-body numerical simulation described in section 2 is precisely to determine the limits of that model and quantify the so-called *finite-temperature effects*.

Nevertheless, it is possible to go further without the help of the simulation under a very simple assumption. Let $\rho(E, t)dE$ be the number of trapped atoms with energies between E and $E + dE$ at time t . $N(E, t) = \int_0^E \rho(E', t)dE'$ is the number of atoms in the trap of energy E or lower. The assumption is that:

$$\frac{dN}{dt}(E, t) = -\Gamma(E)N(E, t) \quad (1.44)$$

Where Γ is a function of E only.

Said another way,

$$N(E, t) = N(E, 0)e^{-\Gamma(E)t} \quad (1.45)$$

which simply means the population of a trap of depth E will decay exponentially at the rate $\Gamma(E)$.

This is reasonable considering that loss curves observed both in experiment and in simulation are very well fit by exponentials, but it is by no means obvious: if the energy distribution in the trap is changing, couldn't the loss rate change too? After all, hotter atoms see a lower effective trap depth and are lost more easily from the trap. Nevertheless, based on our experimental observations, this is a good hypothesis.

Differentiating this with respects to E yields the following equation for the energy distribution:

$$\frac{dN}{dE}(E, t) = \frac{dN}{dE}(E, 0)e^{-\Gamma(E)t} - t \frac{d\Gamma}{dE}(E)N(E, 0)e^{-\Gamma(E)t} \quad (1.46)$$

$$\rho(E, t) = \left(\rho(E, 0) - t \frac{d\Gamma}{dE}(E)N(E, 0) \right) e^{-\Gamma(E)t} \quad (1.47)$$

If $\Gamma(E)$ and the initial energy distribution are known, the energy distribution in the trap can be calculated for all times. In our case we can take $\Gamma(E) = n_b \langle \sigma_E v \rangle$, where $\langle \sigma_E v \rangle$ is the theoretical loss rate coefficient for a trap of depth E (see section 1.3.1).

2 Simulation

As noted at the end of the previous section, the approximation of static trapped atoms cannot adequately describe an ensemble of atoms that is being heated. This prompted us to write a simulation that could precisely describe heating in a magnetic trap.

2.1 Goals and starting hypotheses

Our objective was to create a N-body simulation which reproduced as closely as possible the behaviour of atoms in the magnetic quadrupole trap. Magnetically trapped atom ensembles are very sensitive, hard to manipulate systems for which measurement is destructive as it requires a light to be shone on the atoms. A simulation has the advantage of giving access to information on the system that cannot be easily obtained experimentally.

The simulation is also a way of testing the system's response to a whole range of conditions that cannot necessarily be accomplished in experiments (for example all atoms have the same initial energy). This is helpful when trying to deconvolve various effects.

Moreover, being able to replicate the behaviour of the atoms in the trap is a test of our understanding of the relevant mechanisms at work. The agreement of the simulation with reliable experimental data would serve as a test of its validity.

The effects that were taken into account in the simulation are:

- the classical movement of atoms within the trap
- and the collisions between room-temperature background gas atoms and trapped atoms.

Collisions between trapped atoms were **not** taken into account as they are believed to be negligible in the dilute atomic clouds considered ($n \sim 10^6 - 10^7 \text{ cm}^{-3}$). Thus there is no rethermalization going on (and as a consequence, no evaporative cooling).

2.2 Implementation

Here are some details regarding the way the simulation is carried out. The coding was done in Python and allows the simulation to be run on a cluster.

2.2.1 Potentials

Various trapping potentials can be used:

- a realistic reproduction of the potential felt by atoms in the magnetic quadrupole trap,
- a finite harmonic potential
- and a finite square well.

Another option is to use “frozen” atoms: the trapped atoms have no velocity and are lost only if a collision imparts an energy greater than the trap depth. This last possibility is based on the assumptions made in theoretical work describing the system, where trapped atoms are supposed to be at rest (see section 1.3 and [1, 2, 8]).

2.2.2 Time evolution

For the simulation in a realistic potential, the positions and velocities of particles are updated at each time-step. The algorithm used is the velocity-Verlet or leapfrog algorithm:

$$\mathbf{a}(t) = -\nabla V(\mathbf{r}(t)) \quad (2.1)$$

$$\mathbf{r}(t + \Delta t) = \mathbf{r}(t) + \mathbf{v}(t)\Delta t + \frac{1}{2}\mathbf{a}(t)\Delta t^2 \quad (2.2)$$

$$\mathbf{v}(t + \frac{\Delta t}{2}) = \mathbf{v}(t) + \mathbf{a}(t)\Delta t \quad (2.3)$$

$$\mathbf{a}(t + \Delta t) = -\nabla V(\mathbf{r}(t + \Delta t)) \quad (2.4)$$

$$\mathbf{v}(t + \Delta t) = \mathbf{v}(t + \frac{\Delta t}{2}) + \mathbf{a}(t + \Delta t)\Delta t \quad (2.5)$$

However updating the positions and velocities of every particle at each time-step is time consuming. This is one reason why other, simpler, potentials were introduced. Indeed, if atoms are static or have a constant speed or if the equations of motion have simple analytical solutions (*e.g.* in a harmonic potential), it is possible to “jump” from one collisional event to the next without numerically integrating the trajectory.

2.2.3 Collisions

The occurrence of collisions is a poissonian random process. In the case of the realistic potential, there is a given probability that a collision will occur at each time-step. For the simpler potentials, the interval of time between two collisions is an exponential random variable.

Each time a collision occurs, a function is called to determine the velocity kick received by the trapped particle (or the energy transfer in the case of the “frozen” atoms). The velocity of the trapped particle is known, the velocity of the incoming particle and the collision angle are picked from their respective probability distributions.

The velocities after the collision can thus be obtained by the classical calculation detailed in section 1.2.1.

The differential cross-section for rubidium on rubidium was computed for a number of angles and background particle velocities using a Fortran code written by David Fagnan. Our simulation uses these data to sample scattering angles.

2.2.4 Analyzing results

After each collision, the collision time and the particle’s new energy are stored in a “history” array¹⁰. When the simulation is over, the user can specify a trap depth

¹⁰“Frozen” atoms remain at 0 energy, even after a collision. The energy imparted by the collision contributes to “heating” and is stored in the “history” but the atom does not “keep” it, meaning that in a subsequent collision the atom will still be static.

and the program will then determine if a particle has been lost from the trap, and at what time. With this approach it is possible to obtain a variety of results for different trap depths with only one simulation.

Information that can be extracted from the “history” array includes:

- The loss rate. It is estimated from the loss times using the maximum likelihood estimator for censored data: for an ensemble of N particles with exponential lifetimes, if l particles have been lost at times $t_1, \dots, t_l$ during a time of observation T , the estimate loss-rate is given by [6]:

$$\tilde{\Gamma} = \frac{l}{\sum_{i=1}^l t_i + (N - l)T} \quad (2.6)$$

When using this estimator, we are assuming that the trapped population decays exponentially. This is not obvious, as the loss rate could change over time because of heating (as discussed at the end of section 1.3.3). However fitting exponentials to the decay curves shows that they are indeed exponentials with a very good approximation.

- The energy distribution in the trap at any time, which is essential when studying heating.
- The energy imparted by each collision. In particular, we were able to count “cooling collisions”, which leave the trapped particle with less energy than it had initially. These collisions are rare enough, and involve small enough energy transfers, that they can be neglected.

2.3 Trap dynamics

The simulation can be used to learn more about the dynamics in the trap, independently of collisions. A modified version of the code allows the user to design an experimental sequence in the same way this is done in the experiment: by passing commands such as “ramp coil current”, “wait” or “use RF knife”. The positions and velocities are recorded at each time-step, but there are no collisions.

At first this simulation allowed us to test that the leapfrog algorithm (see section 2.2.2) did indeed conserve the energy of atoms. It also showed that in the trap, the potential energy was approximately twice the kinetic energy, as predicted by the virial theorem (the trapping potential is close to linear).

An important time scale for the system is the time it takes for an atom to “visit” the entire trap. For example the use of the RF knife relies on all atoms above a certain energy crossing the surface defined by the RF in a short period of time. The simulation indicated that this “ergodicity” time scale was on the order of 10 to 100 ms.

3 Experiment

This section briefly describes the experimental setup used to investigate loss and heating in a magnetic trap. More details are given in [15] and [7]. The various types of measurements that were performed and their most significant results are given.

3.1 Apparatus

3.1.1 Laser system

The possibility of trapping neutral atoms in a MOT has been described in the theory section in the simple and imaginary case of a two-level atom. This scheme will work with real atoms on the condition that for the chosen transition the excited state decays only into the ground state. Failure to meet that condition means that some atoms will eventually be driven into *dark* states where they will accumulate. Fortunately such a “closed” loop can be created artificially by optically pumping atoms in the dark states back into the loop.

In the case of ^{87}Rb , the ground state is $5^2S_{1/2}$ $F = 2$ and the excited state is $5^2P_{3/2}$ $F' = 3$ (D_2 transition). They are separated by $\hbar\omega_0 \simeq 780$ nm (near-infrared). The selection rules allow a fraction of the atoms in the excited state to decay into $5^2S_{1/2}$ $F = 1$. Therefore a repump light is added to drive them back up to $5^2P_{3/2}$ $F' = 2$ from which they can decay back into the ground state (figure 9).

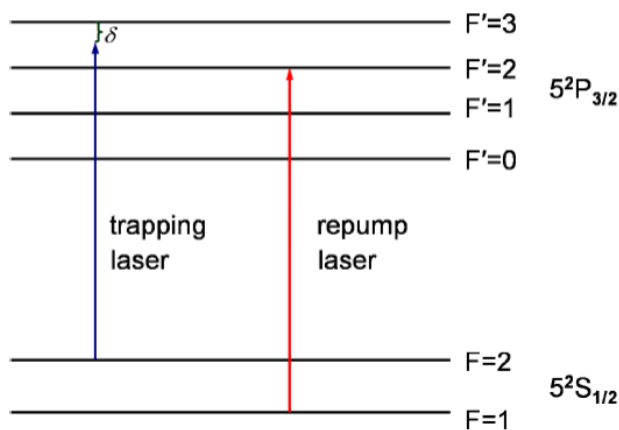


Figure 9: Energy levels for ^{87}Rb and laser transitions used for pumping and re-pumping.

Commercially available diode lasers provide the pump and repump light. The two *master* lasers (one for the pump, one for the repump) are locked to the correct transitions by way of a saturated absorption spectroscopy technique. Their output frequency is stabilized by an electronic feedback loop which controls the position

of a diffraction grating placed outside the cavity. The diffracted light with the desired frequency is re-injected into the cavity, saturating the gain medium and extinguishing all competing frequencies. This scheme provides very stable light with a narrow linewidth, but insufficient power for the creation of a MOT. Therefore this light is injected into *slave* diodes with a much higher power output. Again, injecting light of a given frequency forces the slave to emit at the same frequency [15].

The powerful, stable, narrow-linewidth light can then be driven through acousto-optic modulators (AOM) to adjust the frequency (for example detuning the light for the purposes of trapping). It is then split into three beams and sent in through the vacuum cell along three orthogonal axes. The correct circular polarization is obtained through a set of quarter-wave plates. Mirrors reflect the beams back to complete the MOT (the reflection conveniently changes σ^+ light into σ^- and vice versa).

3.1.2 Test chamber

The test chamber in which the MOT and magnetic trap are formed is a glass cell measuring $6 \times 1 \times 1 \text{ cm}^3$

The ultra-high vacuum required to form a fair sized MOT or a long-lived magnetic trap in that chamber is obtained through several pumps:

- an ion pump which uses combined electric and magnetic fields to ionize the atoms and accelerate them towards a solid electrode
- and chemical getter pumps that rely on chemical reactions to capture nitrogen, hydrogen, oxygen, and methane.

Rubidium vapour can be released into the system by driving a current through a source. The current vaporizes the rubidium atoms contained in an alloy. After refilling for a few minutes, the pressure decreases over the course of approximately 12 hours as the pumps evacuate the extra rubidium atoms. As the pressure decreases the pump becomes less efficient and the pressure ultimately stabilizes.

3.1.3 Imaging devices

The fluorescence from the MOT is collected by a lens and measured using a photodiode. If the MOT density is not too large, this signal is proportional to the number of atoms.

A CCD camera is also used to image the shape of the atom cloud. This is useful when realigning the MOT beams, to make sure the MOT is stable and spherical. Flaring, bouncing and splitting of the MOT must be avoided to ensure a stable signal, with a high signal-to-noise ratio.

3.2 Measuring losses

3.2.1 Experimental sequence

All experiments involving the magnetic trap start by loading atoms from the MOT into the magnetic trap. This is simply done by turning off the lasers and increasing the current in the magnetic coils (from 0.5 A to values between 1.5 A and 15 A). Far-detuning the lasers before turning them off further cools the ensemble, increasing the efficiency of the transfer. If the repump laser is turned off before the pump, the pump will empty out the $F = 2$ ground state and all of the atoms will end up in $F = 1$. Conversely, if the pump beam is turned off first, atoms end up in the $F = 2$ state (figure 9). The experiments described in this report have been performed with ^{87}Rb in the $F = 1$ state. Unless otherwise mentioned, the background gas is a mixture of ^{87}Rb and ^{85}Rb .

The magnetic trap is left to evolve for a given amount of time. The number of atoms left in the trap is then measured by turning the lasers back on and doing a fluorescence measurement. Next, a baseline measurement is made by turning off the magnetic field so that atoms leave the trap but the scattered laser light background signal remains. Finally the MOT is refilled completely and another experimental point can be taken (figure 10).

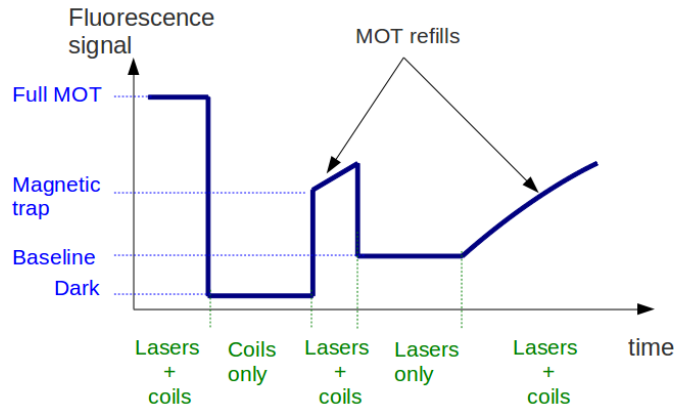


Figure 10: Diagram of a fluorescence curve as observed in a typical experiment.

The population in the magnetic trap is expressed as a fraction of the full MOT population:

$$\text{trapped fraction} = \frac{\text{magnetic trap fluorescence} - \text{baseline}}{\text{full MOT fluorescence} - \text{baseline}} \quad (3.1)$$

3.2.2 Rate coefficients

Since each trap depth U is linked to an angle θ_{min} (equation (1.34)), the dependence of the rate coefficient $\langle\sigma_{UV}\rangle$ on trap depth informs us on the angular dependence

of the differential cross-section [12, 13].

If X refers to a species present in the background vapour,

$$\Gamma = n_X \langle \sigma v \rangle_X + \sum n_i \langle \sigma v \rangle_i \quad (3.2)$$

where the first term corresponds to losses caused by collisions between trapped rubidium and background X and the second term refers to collisions with other gases that might be present in the background vapour (typically H_2 , CO , CH_4 and N_2).

The rate coefficient $\langle \sigma v \rangle_X$ can be obtained as the slope of the plot of loss rate versus X density. Previous work at QDG involved measuring $\langle \sigma v \rangle_{Ar}$ versus trap depth for rubidium-argon collisions. Argon was leaked into the system and a residual gas analyzer (RGA) was used to determine its pressure. For a number of trap depths, loss rates were measured at various argon pressures. Very good agreement with the theory was found [8].

However, rubidium pressure cannot be measured directly with an RGA. Indeed, rubidium atoms tend to stick to the stainless steel walls of the vacuum system. As a consequence, pressure gradients form in the system, making a measurement of the pressure in the cell very difficult.

A possible proxy is the MOT's loading rate, which is proportional to the background rubidium pressure: $R = \alpha n_{Rb}$. It can easily be fitted from fluorescence curves as it is the initial slope of the MOT loading curve (equation (1.41)). The downside is that α is unknown and is liable to change when the MOT beams are readjusted, an operation that must be done nearly daily. Nevertheless, by vaporizing rubidium into the chamber and measuring loss rates (for various trap depths) as the pressure drops back down, one can obtain plots of the loss rate versus R , the slopes of which are $\langle \sigma v \rangle_X / \alpha$. Finally we have the desired result, but scaled by a factor α . This means that when comparing with theory, α remains as a fitting parameter (figure 11).

Note that once α is known, a single MOT loading curve suffices to know the pressure. Unfortunately, as we have mentioned earlier, α is liable to change from day to day. But there is another, simpler way of measuring background gas pressure.

3.2.3 Measuring pressure and trap depth

Equation (3.2) links loss rate, background gas density and trap depth (the rate coefficient corresponds to a unique trap depth). If we are confident that theoretically calculated and experimentally measured rate coefficients agree, a loss rate measurement can be used to determine pressure (if the trap depth is known) or trap depth (if the pressure is known).

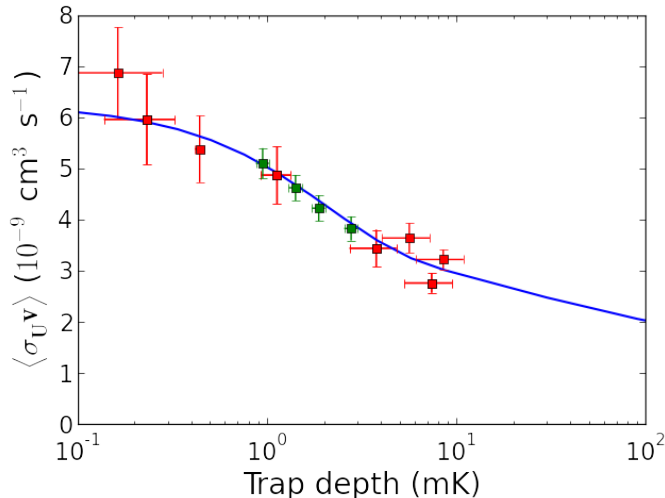


Figure 11: Loss rate coefficient $\langle \sigma_U v \rangle$ versus trap depth U . The blue line is the theoretical prediction for 300 K rubidium atoms colliding with static rubidium atoms. The squares are experimental points, they have been scaled to best fit the theoretical curve; the red and green points have been scaled separately.

Indeed, if the loss rates at various (known) trap depths are plotted against the corresponding theoretical rate coefficients, according to equation (3.2) the points should line up¹¹ and the slope is the background rubidium density. The y-intercept corresponds to the contribution of other gases to the loss rate.

Conversely, if the loss rate is measured for various pressures of X at a constant, unknown trap depth, the slope of the corresponding curve is $\langle \sigma v \rangle_X$, which gives us the trap depth. Note that this technique is very robust and works for all kinds of traps, as demonstrated by Janelle Van Dongen's work at QDG [16].

3.3 Measuring heating

3.3.1 First attempts

A first order approach to the measurement of heating is to divide the trap into two energy bins and count the number of atoms in the lower and higher bins over time.

Consider a trap of depth E_1 , and E_0 an intermediate energy. Bin 0 covers energies from 0 to E_0 and bin 1 goes from E_0 to E_1 . They contain respectively N_0 and N_1 atoms. The heated fraction is defined by:

$$H_F = \frac{N_1}{N_0 + N_1} \quad (3.3)$$

Experimentally, heated fraction measurements start by emptying the upper bin

¹¹If $\sum n_i \langle \sigma v \rangle_i$ does not vary too much with trap depth, which is usually the case.

by setting the RF to $\nu_0 = E_0/h$ ¹². The trap is then left to evolve for some time (the trap depth E_1 is often set by using the RF knife at $\nu_1 = E_1/h$ during that time). At the end of the run, either the RF is set at ν_0 once more, or it isn't. In the former case, the number of remaining atoms is N_0 , in the latter it is $N_0 + N_1$.

At first the frequency of the RF knife was swept between the desired cutting frequency and a high frequency, so as to empty out the high energy atoms faster. However this method gave noisy results, possibly because some atoms were ejected from the trap and then flipped back in. Keeping the RF knife at a constant frequency and waiting for the atoms with enough energy to cross the shell it defines yields far better results. Actually, the precision of our measurements was enhanced to the point that we could abandon the heated fraction technique and sample the entire energy distribution instead.

3.3.2 Probing the energy distribution

Using the RF at a frequency ν after having held atoms in the trap for a period of time t will leave you only with atoms with energies below $h\nu$. This is precisely the cumulative energy distribution (non-normalized) evaluated in $h\nu$, *i.e.* $N(E = h\nu, t)$ if we reintroduce the quantities defined in section 1.3.3. By varying ν , the entire cumulative distribution can be sampled (figure 12).

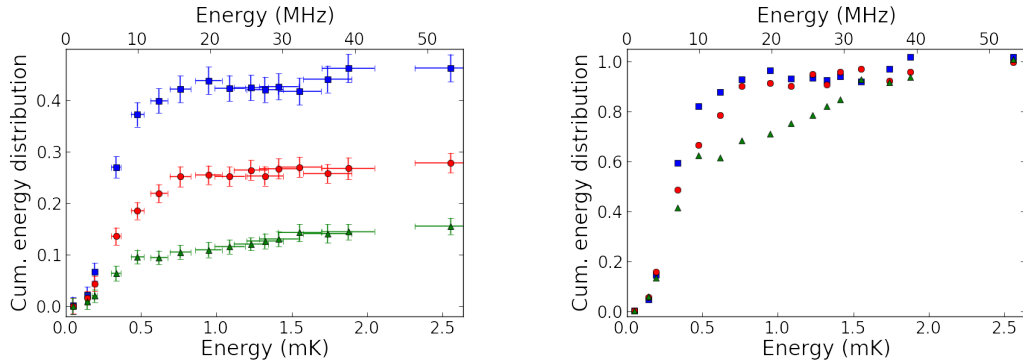


Figure 12: Cumulative energy distribution in a 55 MHz deep trap after 0.4 s (blue squares), 4.5 s (red circles) and 10 s (green triangles); either non-normalized (left) or normalized (right). Not only do the non-normalized distributions get smaller as atoms are lost from the trap, but their shape changes over time, as clearly shown by the normalized distributions. The normalized curves shift towards larger energies as time goes by, indicating heating.

The average energy per particle is $\overline{E(t)} = \frac{1}{N(U, t)} \int_0^U E \rho(E, t) dE$. An integration by part gives a convenient way of obtaining that energy from the data:

$$\overline{E(t)} = U - \frac{1}{N(U, t)} \int_0^U N(E, t) dE \quad (3.4)$$

¹²This step is not absolutely necessary, its object is only to make heating more visible.

If the energy distribution is measured after various wait times, one can plot the average energy versus time. This curve appears to be linear, though we expect it to curve off for longer times¹³. Its slope is what we will call the heating rate $\frac{d\bar{E}}{dt}$.

As expected, the heating rate increases linearly with background rubidium density (figure 13). The slope of these curves is what we will refer to as density-normalized heating rate ($\frac{1}{n_b} \frac{d\bar{E}}{dt}$). It is the equivalent, for heating, of the loss rate coefficient.

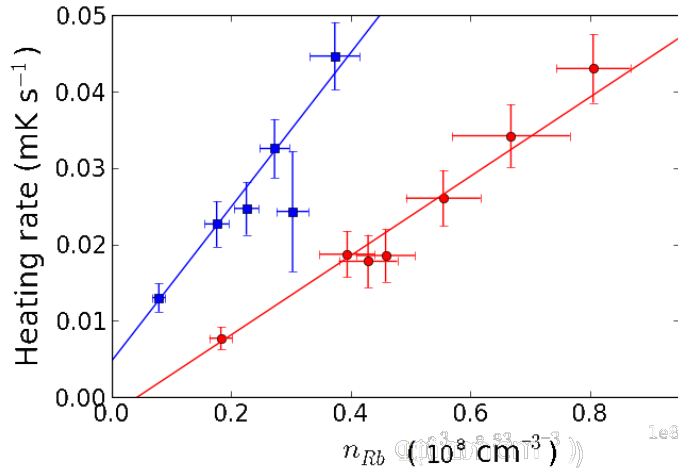


Figure 13: Heating rate $\frac{d\bar{E}}{dt}$ versus background rubidium density n_b for a 40 MHz trap (red circles) and a 60 MHz trap (blue squares). The lines are weighted linear fits. Heating increases linearly with the collision rate, and so with the background gas density. This increase is faster for a deeper trap, as more collisions lead to heating, and less to loss.

As shown by figure 14, larger trap depths allow more heating as fewer collisions lead to loss.

To observe the effects of the atoms' finite temperature on heating the initial temperature of the atoms was varied. The initial energy distribution of the trapped atoms can be changed by using the RF knife to cut out the hotter part of the distribution, leaving the remaining atoms with less energy on average. Another possibility, suggested by the simulation, is to vary the length of time during which the coil current is being ramped when the atoms are loaded from the MOT into the magnetic trap. If the magnetic field is turned on slowly after the lasers are turned off, the atoms have the time to move away from the centre of the trap and end up with more potential energy when the magnetic field is fully turned on, thus

¹³This is a result of the simulation, but it is obvious that heating cannot go on indefinitely as the trap has a finite depth.

the distribution is hotter. On the contrary, ramping the magnetic field quickly leads to a cooler initial distribution (figure 15).

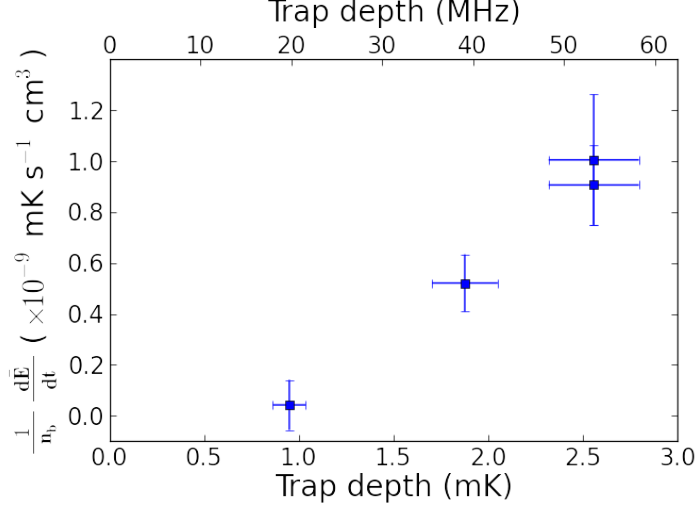


Figure 14: Density-normalized heating rate $\frac{1}{n_b} \frac{d\bar{E}}{dt}$ versus trap depth U . Shallower traps allow less heating to take place.

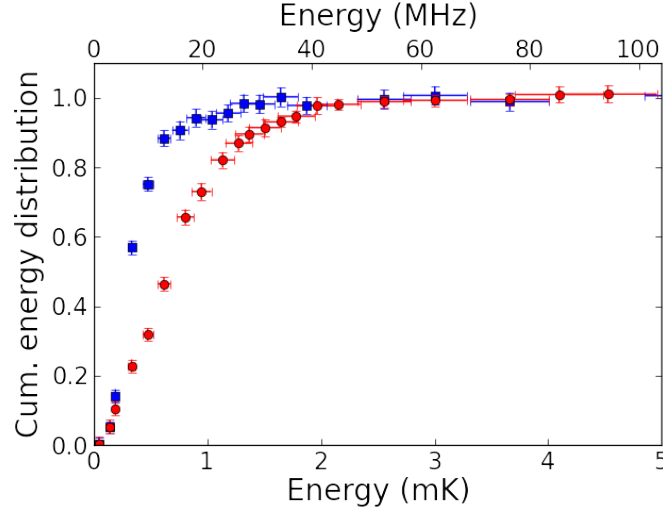


Figure 15: Cumulative energy distributions: either the magnetic field is cranked up instantly (blue squares), in which case the average energy is 8.5 ± 0.3 MHz, or it is ramped up in 50 ms (red circles) and the average energy is 15.4 ± 0.3 MHz. The faster the magnetic field is turned on, the less times atoms have to wander away from the centre of the trap, and the less potential energy they end up having.

Figure 16 shows that a trap that initially holds hot atoms cannot be heated as much as a trap with a colder initial distribution .

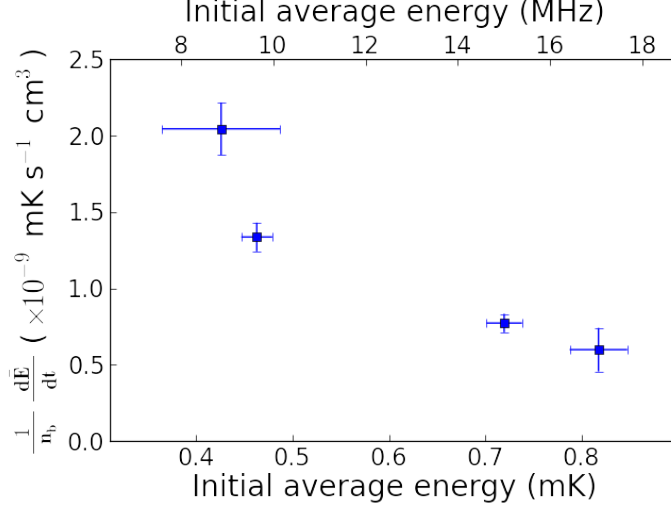


Figure 16: Density-normalized heating rate $\frac{1}{n_b} \frac{d\bar{E}}{dt}$ versus the initial average energy per atom. A warmer distribution heats up more slowly as atoms are more easily expelled from the trap.

Conclusion

Over the course of this internship, my colleagues and I have experimentally characterized loss and heating for an ensemble of cold atoms colliding with room-temperature atoms. In particular, we probed the energy distribution in the trap using an RF knife and were able to detect its evolution over time.

Concurrently, we designed a direct simulation of the trap. Unlike the existent theoretical models, the simulation takes the finite temperature of the trapped atoms into account. Both the experiments and the simulation showed that it was necessary to take finite temperature effects into consideration to describe heating precisely.

Comparison between numerical and experimental is currently under way. We are hoping the two will concur and validate our approach.

Future work at QDG could involve:

- using argon as a background gas in order to have better control over its pressure,
- trying to replicate these results with ^{85}Rb ,
- probing the density distribution of the trapped atoms by using the RF knife in very short pulses, so as to take out atoms at a given position only,

- investigating losses from the MOT due to collisions between ground and excited state atoms,
- linking loss rate and capture rate for a MOT,
- studying inelastic or reactive collisions,
- ...

Research with cold atoms is a very active and fast-moving field. We hope that our results can help others to better deal with limitations such as losses and heating, but we would also like to point out that they are not only hindrances, they are a rich subject of research in themselves.

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